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AN ECONOMIC ANALYSIS OF THE USE OF EXPENDABLE LIGHT-WEIGHT AIR-LAUNCHED PRACTICE TORPEDOES IN THE U.S. NAVY

by

Douglas Wilkinson Payne

Thesis submitted to the Faculty of the Graduate School
of the University of Maryland in partial fulfillment
of the requirements for the degree of
Master of Arts
1968

Thesis
P278

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APPROVAL SHEET

Title of Thesis: An Economic Analysis of the Use of Expendable Light-Weight Air-Launched Practice Torpedoes in the U.S. Navy

Name of Candidate: Douglas Wilkinson Payne
Master of Arts, 1968

Thesis and Abstract Approved: J. Lawrence Hexter
J. Lawrence Hexter, Ph.D.
Assistant Professor of Economics
Department of Economics

Date Approved: October 2, 1967

Thesis and Abstract Approved:
Neil S. Weiner
Program Analysis Division
Institute for Defense Analyses

Date Approved: October 2, 1967

VITA

Name: Douglas Wilkinson Payne

Permanent Address: 1 West University Parkway
Baltimore, Maryland

Degree and date to be conferred: Master of Arts, 1968

Date of birth: October 13, 1930

Place of birth: Baltimore, Maryland

Secondary education: Baltimore Polytechnic Institute, June 1948

Collegiate Institutions attended:	Dates	Degree	Date of Degree
U.S. Naval Academy	6/48-6/52	B.S.	June 1952
U.S. Naval Postgraduate School	8/61-6/62	-	-
University of Maryland	6/66-8/67	M.A.	January 1968

Major: Economics

Minor: Defense Policy

Positions held: Commissioned Ensign, USN on 6 June 1952
Designated a Naval Aviator on 12 February 1955
Present rank is Commander, U.S. Navy

ABSTRACT

Title of Thesis: An Economic Analysis of the Use of Expendable Light-Weight Air-Launched Practice Torpedoes in the U.S. Navy

Douglas Wilkinson Payne, Master of Arts, 1968

Thesis directed by: J. Lawrence Hexter, Ph.D.
Neil S. Weiner

This thesis develops and exercises the methodology to assist the decision-maker in evaluating the present day concept of air-launched torpedoes in the U.S. Navy from an economic standpoint. It investigates the present recoverable torpedo doctrine with a view toward the most effective allocation of resources.

The paper examines characteristics of the present system and that of an alternative system of expendable practice weapons as well as a third alternative of mixed types of practice weapons. Results of the analysis are presented as total cost vs effectiveness and marginal cost vs effectiveness curves. This provides the decision-maker with the comparisons he requires as a basis for selecting the most economically efficient of the alternatives for different levels of output or effectiveness.

Sensitivity tests are carried out to determine the effect of variations in major parameters and assumptions upon the study results.

The study results provide to the decision-maker the most efficient choice available to him over the spectrum of output but does not provide data as a basis of an "optimum" solution. Such an optimality decision can only be determined by the decision-maker himself with the higher level political or military considerations that he brings into the problem. A cost benefit analysis must have limited objectives. It is designed to provide the decision-maker with the best information available so as to make his optimality solution a correct one.

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CHAPTER I

INTRODUCTION

A major and continuing problem in any military weapon system is the manner in which peacetime weapon proficiency is maintained so that in wartime the weapon will provide the military power it is designed to produce. Submarines may be attacked by aircraft, surface ship or by other submarines in a variety of ways, but in all cases a primary offensive anti-submarine weapon is the torpedo. Air and surface units both use the identical torpedo in anti-submarine warfare (ASW) operations.

In order to maintain a weapons proficiency and wartime firing capability in submarine attack, naval units practice in peacetime with a recoverable "exercise" torpedo. The exercise torpedo is identical to the wartime torpedo, except that the explosive head or warhead is removed and replaced by an inert exercise head of equal buoyancy. Regardless of the type torpedo or the type launching vehicle, the basic concept of a recoverable and reuseable weapon remains. Air launched torpedoes are recovered by Torpedo Retrievers (called "TR" boats) or by destroyer or destroyer-type ships. Torpedoes launched by surface ships or submarines are picked up by the firing ship herself or by other TR or destroyer-type vessels.

The Navy faces a similar consideration in the use of these practice torpedoes as does a firm attempting to maximize profits subject to a budget or resource constraint. It is doubtful that the Navy has considered this problem within the analytic framework of an economic analysis. Being economic does not mean doing the objective regardless of cost, or conversely it does not mean reducing expenditures regardless of the objectives. Economics is concerned with allocating resources--resources that are in scarce supply and have conflicting demands upon their utilization.

The Navy has always recovered her exercise torpedoes. The nature of the recovery process, however, imposes some latent cost on the training received. Basically, the requirements are for daylight, good visibility and relatively calm seas in order to permit a safe recovery of the weapon and such conditions are the antithesis of expected ASW conditions in wartime. As a result pilots and destroyermen remain unpracticed and untested under the most probable attack conditions. This is the type environment in which the entire weapon system must provide its maximum designed military effectiveness.

The costs associated with the concept of torpedo recovery and reuse are high and in some cases hidden. For example, the investment cost of each of the Navy's TR boats and associated on-board equipment is one-half a million dollars. The cost to refurbish a single torpedo is $900 and even under ideal launch conditions some 20 per cent of the torpedoes fired are expended or lost anyway.[1]

---

[1] Quarterly Status of Torpedo Firings, U.S. Naval Underwater Weapon Research and Engineering Station, Newport, Rhode Island, 1963-1966.

Because of the apparent cost and operational disadvantages of the present recoverable torpedo concept, it seems well worth investigating an alternative expendable-type torpedo to see what such a concept might offer in reduced cost and increased operational advantages. This paper will address an approach to a cost effectiveness analysis comparing the present air-launched torpedo to a comparable expendable torpedo. A relationship between output (squadron combat capability), and the number of torpedo firings conducted, will be tested as a measure of effectiveness (MOE) of torpedo utilization in terms of squadron capability. A discounted stream of costs for the present torpedo and the expendable alternative at different firing levels will be determined so that total cost and marginal cost curves can be derived.

CHAPTER II

METHODOLOGY

Basic Procedure

The basic methodology followed is the consideration of the torpedo problem as one of scarce resource allocation analogous to the selection of the least cost production process of the firm in producing a product, its output. For the profit maximizing firm, the problem is approached by using the firm's production function which relates output levels to levels of factor inputs for each production process. Given this production function and the cost relationship for the factor inputs, a function can next be derived which gives the total cost associated with various levels of output. A comparison of these total cost curves for each production process demonstrates which process is the most "efficient", that is, the least total cost for a particular level of output. In order for the firm to determine its "optimum" level of output, the marginal cost and consumer demand functions are required. The firm interested primarily in maximizing the difference between output and cost (i.e. profit maximization) determines its optimum level of output at that production point at which marginal cost and marginal revenue are equal.

In attempting to evaluate military systems in this manner, two problems are present. First, the military system has no market-type mechanism for the determination of a consumer demand function. Second, although costs can be measured in dollars, output or effectiveness generally cannot be. The concept of profit maximization has no meaning if output cannot be measured in terms of dollar values as is cost. This cannot be done in the vast majority of military systems.

At this point the analogy for the determination of resources allocation for defense systems and the private sector differs. To determine optimal output the firm equates marginal cost and marginal revenue, but the military system (having no easily derived or expressed demand curves) must depend upon value judgments of a higher level decision-maker to determine the level of output. In economic analysis, the military analyst can present the total cost and marginal cost relationships so that for any desired level of output the most efficient alternative can be determined. Stated equivalently, at any predetermined output the least-cost alternative can be selected. Selection of either the desired cost level or the desired output is an input following the analysis that must be provided by the higher level decision-maker. Most often the military systems analyst must be satisfied to provide to the decision-maker the spectrum of costs at various output levels so that the decision-maker will have the best available information for his choice.

The basic methodology in step form is as follows:

1. Determine a production function relating input to a measure of effectiveness or output.

2. Select alternative production processes; alternative methods of producing the same output.

3. Compute system cost streams for each alternative solution in terms of resource costs.

4. Develop Output vs Total Cost relationship for each alternative and Output vs Marginal Cost relationship for each alternative.

5. Due to the uncertainties in many of the assumptions, conduct a series of sensitivity tests to ascertain the significance of variations in the analysis parameters.

Before examining these steps in more detail, it would be wise to digress to be certain the problem contains the essential element of an economic analysis. These elements are described by E.S. Quade[1] as follows:

> "1. The objective. Systems analysis is undertaken primarily to suggest, or at the very least, to help choose a course of action. This action must have an aim or objective. Policies or strategies, forces or equipment are examined, compared, and preferred on the basis of how well and how cheaply they can accomplish the aim or objective.
>
> 2. The alternatives. The alternatives are the means by which it is hoped the objectives can be attained. They need not be obvious substitutes or perform the same specific function.
>
> 3. The costs. Each alternative means of accomplishing the objectives implies the use of specific resources which cannot then be used for other purposes.

---

[1] E. S. Quade (ed.), Analysis for Military Decisions (Chicago, Illinois: Rand McNally and Co., 1964) p. 155.

4. The model. The model is a representation of the situation under study designed to predict the cost and performance of each alternative. It abstracts the relevant features of the situation by means which may vary from a set of mathematical equations or a computer program to an idealized description of the situation in which judgment alone is used to assess the consequences of various choices.

5. A criterion. A criterion is a rule or test by which one alternative can be chosen in perference to another. It provides a means for using cost and effectiveness to order the alternatives."

With these concepts in mind, we can describe the torpedo problem in these same terms.

The objective is the highest combat effectiveness of the Navy's anti-submarine aircraft squadrons for a given cost. This means to attain the highest capability in the ASW squadron to carry out its assigned mission which is: To search, locate, identify, track, and if required, destroy submarines.

The alternatives selected to meet the objective are two types of torpedo concepts: The recoverable torpedo (referred to hereafter as REC), and the expendable torpedo (referred to hereafter as EXP).

The costs incurred by each alternative are substantiated in Chapter IV.

The model is determined in Step 1 and Step 3 of the basic methodology. The production function and cost equations constitute the model.

The criterion will be established in Step 4 and Step 5 with the development of total cost and marginal cost curves and the sensitivity analysis. This will provide the decision-maker with the level of cost for the range of effectiveness for each alternative.

Let us return now for a closer examination of the five steps of the methodology.

1. The production function. What is ideally desired is the relationship between some measure of effectiveness and resource inputs. The resource input in this case is the number of torpedoes dropped in peacetime practice exercises. The measure of effectiveness (MOE) must fulfill two basic requirements. It must be first, relevant to the mission of the system being considered, and second, measurable.[1] The MOE desired is an ASW squadron's combat capability which is not a measurable factor under most peacetime conditions. The MOE which was selected as a suitable substitute is the measure of an ASW squadron's peacetime capability; the squadron readiness score. These readiness scores are primarily based upon pilot performance in various anti-submarine exercises and are extremely competitive since they are the basis for annual squadron efficiency awards. How peacetime readiness scores relate to squadron combat capability is not known. For purposes of the study, a one for one correspondence will be assumed without damage to the basic concepts involved in the analysis.

---

[1] William A. Niskanen, U.S. National Security Objectives and the Choice of Measures of Effectiveness, Internal Note N-301(R) (Arlington, Va.: Institute for Defense Analyses, 1965) p. 4.

2. Alternatives. As discussed previously, the scope of the analysis has limited the alternatives to two basic concepts. The recoverable air-launched torpedo concept (REC) compared to the expendable torpedo concept (EXP).

3. Cost streams. The systems costs for each alternative will be fully discussed in Chapter IV.

4. Choice criterion. The curves of Readiness (Effectiveness) vs Total Cost and Readiness vs Marginal Cost are developed to provide to the decision-maker the best information available to assist him in decisions on output level. The total cost curves will demonstrate which system is least costly at each level of output. The decision-maker then may select the most efficient system subject either to a cost constraint (i.e. maximizing effectiveness for a given budget) or an effectiveness constraint (at a given effectiveness, minimizing the dollar cost). Both approaches are analytically equivalent.

Marginal cost curves provide to the decision-maker the incremental costs associated with incremental changes in the effectiveness measure. While marginal cost information is not directly used in the determination of efficiency, it is an important adjunct to the total cost curves and is useful in examining effectiveness comparisons with other Naval units performing similar missions.

5. Sensitivity. Once the total and marginal cost curves are derived, the curves must be analyzed with regard to their sensitivity to value judgments involved in key input parameters and assumptions. Sensitivity tests are conducted on the following parameters:

a. Planning period
b. Discount rate
c. Basic torpedo cost

Basic Assumptions

In view of the importance to the analysis of the basic assumptions, they are discussed in the final section of methodology.

In any analysis one of the primary driving forces is the initial assumptions made for the study. It is vital, therefore, that the assumptions be made clear at the onset. If the real world of the decision-maker dramatically differs from the scenario of the study, he is better able to adjust for these differences knowing precisely what the initial study assumptions are.

For the purposes of the study, the REC torpedo consists of a head, control section and afterbody section. During peacetime training exercises, the explosive head is replaced with an exercise head which contains telemetry equipment to determine the precise run of the torpedo, whether or not it acquired and hit the target, and a mechanism for surfacing the torpedo at the completion of its run. The air-launched weapon must be accurately placed in the water by the launching aircraft relative to the submarine position. It is parachute retarded, actively (with sonar) hunts the submarine in a helical pattern, and once acquisition of the target is established, homes in on the target. Characteristics, performance and reliability of the expendable alternative (EXP) are assumed to be comparable to the REC.

Trade-offs between these two alternatives are readily apparent. The EXP has no launch restrictions so that it can be used in any ASW training exercise, under any conditions. This increase in training realism should provide some increase in pilot attack capability, although such an increase is accepted as an immeasurable quantity. The REC has rather severe launch restrictions. It is limited by the relatively calm sea conditions necessary for a safe torpedo retrieval. Conditions of daylight and an on-the-scene-recovery vessel are two additional requirements. Other trade-offs exist due to the fact that the REC can monitor its own run and provide performance information which is useful in design and performance trouble-shooting. The EXP cannot. In order to alleviate this apparent drawback in the EXP, let us examine the EXP further.

The EXP concept involves the production of a weapon for ASW in much the same manner as a round of ammunition. Since it would be constructed for one firing only, cost savings are anticipated following a lessening of material tolerance and strength requirements that go into such parts as its power plant and guidance sections. A shell of equal buoyancy, for example, could be substituted for the costly exercise head. Throughout the weapon, savings could be expected by substituting cheaper materials perfectly suitable for a one-shot practice weapon which is not intended for repeated uses as is the REC. No change in basic torpedo design is assumed, so that a minimal additional research and development expenditure is required.

The weapon would be completely assembled for launching at the factory, and except for shelf tests would require no special tests prior to launch. Despite the reductions in cost per torpedo that would result, additional costs would be incurred in order to provide a sample testing of the production output to maintain torpedo reliability at the approximate operational level presently existing for the REC.[1] Such test and evaluation would be above that level of testing presently required with the REC.

Hit information provided by either weapon is approximately equivalent. In the REC the monitor telemetry equipment confirms the hit or confirmation comes from actual target contact. The EXP hit confirmation is by actual contact only. While this restriction will prevent use of the EXP on a target not fitted with baffles to protect the ship's screws, this same restriction is in effect for REC utilization today.

---

[1] The precise torpedo reliability figure is classified but is assumed as 80 per cent for purposes of the analysis.

Thus, there are relevant trade-offs between operational restrictions and between torpedo monitor information for the two types of torpedoes. While these trade-offs must be considered by the decision-maker, they do not enter into the effectiveness calculations nor the costing data. Any bias from these assumptions is intended to be in favor of the present system.

Additional assumptions made for the analysis are as follows:

1. Total cost of the REC is $15,000 which includes basic procurement costs of the entire exercise configured torpedo (exercise head, main assembly, and accessories). Although the precise cost of procurement is classified, the $15,000 figure is sufficiently accurate for the analysis.

2. Total procurement cost for the EXP is $10,600. This assumption is further clarified in the chapter on cost.

3. A failure rate (resulting in torpedo loss) will be assumed to be 20 per cent for the REC. In the case of the EXP 20 per cent of the firings will be considered no-hit failures due to torpedo reliability.

4. No costs are incurred for destroyer-type recovery of torpedoes; that is, that under the present system, a surface or air firing which is recovered by surface warships is done at no dollar cost due to the fact that the assumed opportunity costs for the warship and her manpower and equipment for a one-hour recovery operation are zero. Recovery costs are allocated to TR boat pick-up only, and

these costs are fixed costs independent of the level of firings. At no firing levels considered feasible is the TR capacity exceeded and the purchase of additional TR boats required. The life span of a TR is 15 years so that TR replacement is assumed minimal. A capital expenditure of $500,000 a piece is required for replacement of two of the thirty craft each year.

5. No additional REC torpedoes are purchased during the planning period. This is not as limiting an assumption as it might first appear since the present stockpile will permit annual total firing levels of 4500 torpedoes for 10 years, which is well above anticipated needs.

6. REC torpedoes remaining in the inventory on the completion of the planning period have a salvage value based upon exponential decay.

7. Hit data provided by telemetry in the REC is provided by the target in the case of the EXP. Run data also provided automatically by the REC is assumed unnecessary for the EXP since the concept accepts the increased costs associated with a test and proof program prior to delivery to maintain the 80 per cent reliability figure of the REC.

8. The EXP is assumed to require only limited research and development costs since no changes in torpedo technology are called for. Those research and development costs required are considered incurred in the first year of the planning period.

9. The REC is a joint weapon for both aircraft and ASW surface ships. Any future weapon is likely to be a joint weapon also. For this reason a constant level of surface firings (destroyer type) are assumed throughout the analysis and the level of aircraft firings is allowed to vary on top of this surface firing level. This is true for all alternatives considered.

10. This study used the three basic types of aircraft squadrons that are employed in ASW work. In the Atlantic Fleet data on all three types was available and was used. These types are:

    1. VPL - Atlantic-based patrol plane squadron
    2. VSL - Atlantic-based carrier ASW squadron
    3. HSL - Atlantic-based helicopter ASW squadron

    In the Pacific Fleet data on only two types of squadrons was available. These types are:

    1. VPP - Pacific-based patrol plane squadron
    2. VSP - Pacific-based carrier ASW squadron

    These five categories are hereafter referred to as data bases.

CHAPTER III

MEASURE OF EFFECTIVENESS

The establishment of a valid production function is one of the most difficult steps of the analysis. Here must be specified the relationship between input to the process and output. Ideally, larger amounts of squadron training would result in higher squadron ASW capability and the more complete and realistic the training, the greater the impact of training on capability. The culmination of an ASW exercise in a "simulated" tropedo drop would seem to lose some measure of its reality and training value, since the exercise results are critically dependent upon how the pilot conducts his attack on the submarine.

The output measure of an ASW squadron is taken to be the peacetime "readiness" score. Here squadrons are evaluated on their ability to carry out their mission by a common senior and are under a strong competitive stimulus each year for an award based on these readiness scores. Although inputs include such diverse items as the numbers of plane commanders, the amount of day and night carrier landings, and the amount of instrument hours and total flight hours, the score is primarily based upon operational performance of the squadron in certain competitive exercises and, therefore, has a strong operational flavor.

The thesis of the paper concerns the construction of this production function--what inputs in relation to torpedo expenditures are significant in the determination of the readiness output of an ASW squadron. Stated differently, to increase squadron readiness, what would be the anticipated new torpedo drop rate associated with the change?

In order to determine the strength of the relationship that exists between readiness and torpedo expenditures, a statistical analysis was conducted on various combinations of the following independent variables:

X_6 = per cent torpedo hits per squadron

X_7 = log X_6

X_8 = number of torpedo firings per squadron per year

X_9 = log X_8

X_{10} = number of torpedo firings per squadron per aircraft per year

X_{11} = log X_{10}

The analysis was performed to determine what influence these variables have on squadron peacetime readiness scores. Both semi-log and total-log type regressions were computed using data for the 3-year period, September 1963 through September 1966, on land-based and carrier-based ASW fixed-wing and helicopter squadrons. The Harvard Economic Research Project HTW Least Squares Regression Program was used.

Dummy variables were inserted in the regression analysis in order to isolate differences in measurement between the Atlantic Fleet Units and Pacific Fleet Units. An obvious bias existed in the readiness figures depending upon type of squadron and geographic locale.

This bias was eliminated by using X_1, X_2, X_3, X_4 and X_5 as dummy variables as follows:

$$X_1 = \begin{cases} 1 \text{ if Pacific-based patrol squadron (VPP)} \\ 0 \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 \text{ if Pacific-based carrier ASW squadron (VSP)} \\ 0 \text{ otherwise} \end{cases}$$

$$X_3 = \begin{cases} 1 \text{ if Atlantic-based patrol squadron (VPL)} \\ 0 \text{ otherwise} \end{cases}$$

$$X_4 = \begin{cases} 1 \text{ if Atlantic-based carrier ASW squadron (VSL)} \\ 0 \text{ otherwise} \end{cases}$$

$$X_5 = \begin{cases} 1 \text{ if Atlantic-based helicopter ASW squadron (HSL)} \\ 0 \text{ otherwise} \end{cases}$$

The HTW Program was run on an IBM 7094 computer. In all, 98 separate regressions were computed (excluding step regressions) on either squadron readiness or the log of squadron readiness as the dependent variable. Separate regressions were run on the data separated by years (1964, 1965, 1966), and separated by type squadron and geographic location (such as patrol planes Atlantic or patrol planes Pacific).

One would expect intuitively for increased torpedo expenditures to result in increased squadron readiness. One would also anticipate diminishing marginal returns in terms of readiness as torpedo expenditure levels are raised. At some level of torpedo expenditures, further increases should result in no appreciable readiness change. Plots of portions of the data confirmed these expectations. For these reasons semi-log and total-log transformations were favored for the regression form in order to assimilate the asymptotic properties of the production function.

Nevertheless, three problem areas in the determination of the production function remain:

1. Misspecification - the form of the relationship may be misspecified.

2. Omitted variables - variables that go to make up the readiness score are admittedly absent in the proposed relationship. The proposed regression, however, is attempting to demonstrate that for a desired change in readiness there should be a necessary corresponding change in the rate of torpedo expenditures associated with the new readiness level.

3. Errors in measurement - although the readiness score may look deceptively precise on the surface, it is composed of the value judgments of (hopefully) impartial exercise observers and is subject to measurement errors.

Five resulting regression equations are listed below:

1. Run 137

$R^2 = .47 \qquad \bar{R}^2 = .43 \qquad R = .67 \qquad df = 93$

$$\text{Readiness} = \underset{(3.3)}{55.1X_1} + \underset{(3.1)}{63.8X_2} + \underset{(2.8)}{65.8X_3} + \underset{(3.0)}{73.6X_4} + \underset{(3.5)}{65.0X_5}$$

$$+ \underset{(2.9)}{.6X_6} + \underset{(2.4)}{10.8X_9}$$

2. Run 211 (without dummy variables)

$R^2 = .16 \qquad \bar{R}^2 = .14 \qquad R = .38 \qquad df = 98$

$$\log \text{Readiness} = \underset{(.011)}{1.8192} + \underset{(.00146)}{.00635X_8}$$

3. Run 221 (without dummy variables)

$R^2 = .14 \qquad \bar{R}^2 = .13 \qquad R = .35 \qquad df = 98$

$$\log \text{Readiness} = \underset{(.014)}{1.8075} + \underset{(.01895)}{.0764X_9}$$

4. Run 251

$R^2 = .45 \qquad \bar{R}^2 = .41 \qquad R = .64 \qquad df = 94$

$$\log \text{Readiness}\ \underset{(.014)}{1.7578X_1} + \underset{(.013)}{1.8199X_2} + \underset{(.013)}{1.8288X_3} + \underset{(.013)}{1.8761X_4}$$

$$+ \underset{(.018)}{1.8238X_5} + \underset{(.0012)}{.0050X_8}$$

5. Run 261

$R^2 = .44$ $\bar{R}^2 = .41$ $R = .64$ $df = 94$

$$\log \text{Readiness} = \underset{(.016)}{1.7440X_1} + \underset{(.015)}{1.8096X_2} + \underset{(.015)}{1.8185X_3} + \underset{(.015)}{1.8652X_4}$$

$$+\underset{(.019)}{1.8141X_5} + \underset{(.016)}{.0632X_9}$$

Neither exogenous variables X_6 or X_7 show significance in the regression analysis. This is believed due to the fact that data consists primarily of instances of squadron drops in a low range. In this range of drops, one additional success or failure generally caused a large proportional change in the per cent of hit figure, hence X_6 tended to fluctuate widely. The coefficient and standard error figures of X_6 in Run 137 are typical and X_6 and X_7 were found not to have a significant determination effect on squadron readiness capability for the data available.

Multicollinearity is present between the explanatory variables X_8 (torpedo firings per squadron) and X_{10} (torpedo firings per squadron per aircraft) due to their close interrelationship. The multicollinearity is demonstrated in Runs 117 and 118 in Appendix A. Run 117 is a regression on X_8 and Run 118 is the next step regression which adds X_{10} to the explanatory variables. Variable X_8 is initially significant in Run 117, but with the addition of X_{10} the level of influence of X_8 changes (the coefficient jumps from .8 to 2.3) and X_{10} assumes a negative coefficient which is not significant at the 5 per cent confidence level. The number of torpedo firings per squadron (X_8) is significant at the 5 per cent level on both runs. The likelihood that X_8 would have a positive influence on readiness and X_{10} have a negative influence in the same expression does not appear reasonable and cannot be intuitively supported. The variable X_8, therefore, is considered a more desirable regression variable than X_{10}.

Regression attempts on the cross-section data separated by year or by squadron type did not prove very satisfactory. The 1964 data shows characteristically low $\bar{R}^2$ values often below .05 and with non-significance of the non-dummy variables. This is probably caused by the dearth of data available for 1964 and the resulting low degrees of freedom. Run 192 in Appendix A is a sample regression of the 1964 data.

The 1965 and 1966 data is characterized by higher R^2 and $\bar{R}^2$ values (in the .6 range), but generally with non-significance of the non-dummy variables. When the variables do demonstrate significance, the R^2, $\bar{R}^2$ values fall to the .2 range. Run 117 in Appendix A is a sample of the regressions obtained on 1965 or 1966 data. Attempted regressions on data by type squadron likewise show poor $\bar{R}^2$ values and non-significance of the variables.

CHAPTER IV

COST ANALYSIS

> "Whenever, for the attainment of a fixed level of an objective, we seek to minimize the drain upon the input resources because they have value to the attainment of objectives elsewhere, we seek to minimize the real costs. By real costs we mean the value of the alternative objectives that are sacrificed."[1]

Since comparative economic real costs cannot be explicitly derived in defense systems analysis, estimated money costs of the two systems are utilized as an effective yardstick of comparison. The computation of dollar cost is used as an approximation of the economic real costs involved in aggregating the systems cost of the alternatives.

The following general rules for the treatment of time distributed expenditures are used in the cost analysis phase:

1. The initial year of the planning period is the first year that expenditures for the torpedo system might be influenced by the analysis. Fiscal 1969 is the initial year of the planning period. The choice of a planning period is primarily determined by the expected service life of the system under consideration. The expected service of a torpedo varies considerably, but in the past torpedoes have remained in service use anywhere from 10 to 15 years or longer. A planning period of 10 years is

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[1] Malcolm W. Hoag, Operations Research, Vol. 4, No. 4, August 1965, p. 448.

assumed for the REC and EXP torpedoes in an attempt to minimize the uncertainties of end period values.

2. Any assets available at the beginning of the planning period are valued at their highest marketable value in an alternative use. Similarly, any assets available at the end of the planning period are valued at their value in continued operation or in some alternative use, whichever is higher.

3. Whatever expenditures are necessary to maintain the physical condition of assets and to provide replacements for expected attrition are added to the operating costs in the years the expenditures are made or anticipated.

4. The salvage value of a military asset maintained in a constant physical condition throughout the planning period is based on a constant exponential decay based upon the expected service life of the asset.[1]

5. An interest rate of 10 per cent is used to aggregate the time distributed costs of the program.

In amplification of the procedure of discounting used in the cost analysis, consider the plant manager of a firm with funds under his direct control. Each $100 which he controls today is equivalent to $110 one year from now (assuming a

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[1] William A. Niskanen, A Suggested Treatment of Time-Distributed Expenditures in Defense Systems Analysis, (Internal Note N-396(R), Arlington, Va.: Institute for Defense Analyses, 1966), p. 9.

10 per cent interest rate) and conversely an obligation of $110 a year from now is equivalent to a $100 obligation today.

Similarly, an obligation of $259 ten years from now is equivalent to a $100 obligation today, assuming a constant interest rate of 10 per cent. There are various ways in which the plant manager might invest the presently available funds of the firm. He can obtain at least the going market interest rate for the use of these funds. Thus, in making investment decisions he must consider the expected rate of return as well as the time period in which the investment will take place in determining the total cost to him.

The procedure used to make the future costs occurring at different future times equivalent is to weight future costs according to a fixed interest rate so that all costs can be stated in terms of present value of the dollar. The present value of the stream of dollar costs spent in year t is:

$$\text{Present Value} = \sum_{t=0}^{T} \frac{C_t}{(1+r)^t}$$

where C_t is dollar costs in year t and r is the interest rate. This present value is a measure of the opportunity cost of the funds presently available; that is, the marginal productivity of capital.

The government is in a precisely analogous position to the firm. Some future cost in year 10 of $259 is valued at only $100 today because that same $100 invested today would yield the $259 in 10 years, assuming a constant 10 per cent interest rate. The demand for present capital must be charged at least the rate representing the market evaluation of the marginal productivity of capital.[1]

The interest rate of 10% represents the total cost to the government from government borrowing activity and is somewhat higher than the interest rate on government bonds. The borrowing rate is adjusted for the additional personal income taxes gained on the interest payment and for the personal and corporate income taxes lost from the reduction in private capital formation. This 10 per cent is computed as follows:[2]

Government Borrowing Rate	+4.7%
Personal Income Tax on Interest	-1.6%
Corporate Taxes Foregone	+4.6%
Personal Income Taxes Foregone	+2.0%
	+9.7%

The categories of the cost analysis are broken down as follows:

A. Research and Development: systems development and test evaluation costs.

B. Investment: installations, torpedo procurement cost, initial stock level costs, initial training, command support costs and retreiver ship replacement costs.

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[1] Charles J. Hitch and Roland N. McKean, The Economics of Defense in the Nuclear Age (New York: Antheneum, 1966) p. 206

[2] Niskanen, A Suggested Treatment of Time-Distributed Expenditures..., p. 17.

C. Operating Costs: torpedo attrition, refurbishment, test support, spare parts, training, retriever ship operating costs.

Costing the REC Torpedo[1]

Research and Development costs are sunk costs for the REC torpedo. This is not a bias in favor of the present system as it may seem at first, but is rather a correct procedure for obtaining a rational look at the different weapon systems costs. All research and development work was completed by 1964 and the torpedo is part of a presently operational system with no anticipated future torpedo procurement. Funding for 1967 includes the final purchases of REC torpedoes and they will be continued in service until superceded or the large stockpile is expended.

All assets except the torpedo retrieval boat are considered to have a zero market value and are valued as a sunk cost. The thirty TR boats, however, do have a market value of $250,000 apiece (original cost of $500,000 less specialized equipment) and this $7.5 million is an investment expenditure of the first year of the planning period.

The costing for the REC, then, is composed of investment and operating costs alone and will be explained by means of a cost equation.

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[1] Source of the data for the cost computations is the Quarterly Status of Torpedo Firings Reports, 1963, 1964, 1965, 1966, prepared by the U.S. Naval Underwater Weapons Research and Engineering Station, Newport, Rhode Island; as well as interviews with U.S. Naval Ordnance Systems Command, In-Service Torpedo Group personnel.

Let

C_{REC} = Total amortized system cost for the REC torpedo for the planning period

C_{oj} = Basic operating cost j^{th} year

C_R = Refurbishment cost per recovered torpedo

C_D = Cost of depot maintenance and spare parts

C_S = Cost of torpedo recovery ships (operating costs and replacement costs combined)

C_A = Attrition cost per torpedo not recovered

V_S = Total torpedo retriever salvage value

x = Number of total firings per year during planning period (air and surface vessels)

j = Planning year

r = Interest rate

m = Length of planning period

The cost equation for the REC becomes:

$$C_{REC} = \sum_{j=1}^{m} \frac{C_{oj} + .2x\ C_A + .9C_S + .8x\ C_R + C_D - V_S}{(1 + r)^j}$$

The letters in the equation represent the following cost breakdown. Basic operating costs, C_{oj}, covers the Fleet Exercise Support, limited spare parts requirements, and Proof and Test work to maintain torpedo reliability at a pre-selected level of 80 per cent. For the first year only, C_{oj} includes the investment expenditure of $7.5 million for the torpedo retriever assets of the force. Refurbishment costs per recovered torpedo, C_R, are conservatively estimated at $900 per unit as shown in the table below.

Expendables

Battery	$1000
Parachute	150
Weights, lanyard, squibs & suspension bands	158
Replacement of parts	50
Labor (17.5 man hours)	35
	$1393
Battery salvage	-500
	$ 893 ≈ $900

C_A, the cost of each torpedo attrited is equivalent in dollar value to the estimated initial basic procurement cost of the REC of $15,000. The .2x represents the expected rate of attrition times the number of total firings.

The cost of torpedo recovery ships, $.9C_S$, requires further explanation. As previously stated in the chapter on basic assumptions, no costs are incurred by warship torpedo recovery since realistically the opportunity costs of such vessels, men, and equipment would be zero. C_S represents an aggregate annual operating cost of the 30 TR vessels presently in service, assuming a replacement of two of these vessels annually; that is, a service life of 15 years. The coefficient of .9 recognizes the other uses to which these boats would be subjected along with their normal recovery duties such as rescue and utility requirements.

C_D represents that portion of depot operations costs allocated to the REC and major spare parts requirements and is estimated at $1.5 million annually. C_{oj} and C_D collectively amount to $10 million annually. Table 1 gives a summary of results of the cost analysis for the REC.

Costing the EXP Torpedo

Beyong the basic assumptions of Chapter II certain costing assumptions are required to make the analysis more manageable. These are:

1. EXP basic procurement cost is $10,600.
2. EXP torpedoes are not stockpiled, but only those to be used are purchased each year. Thus salvage value at the close of the planning period is zero.
3. In production procurement an 85 per cent learning curve[1] is realized for annual production levels.
4. Surface launch rate is constant at the same level as for the REC costing.
5. Annual expenditure rate is considered constant throughout the planning period.
6. While REC research and development is estimated at $11.7 million over a nine-year period, little R&D costs for the EXP are expected due to the basic assumption that the EXP is not a redesign concept but merely a less costly version of the present weapon. A cost of $2 million is assumed to cover research and development work for the EXP.

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[1] R. N. Anthony, Management Accounting (Homewood, Illinois: R.D. Irvin Co., 1960), p. 528.

TABLE 1[1,2]

REC TORPEDO COST DATA FOR 5 AND 10 YEAR PLANNING PERIODS AND 5 PER CENT AND 10 PER CENT DISCOUNT RATE

TOTAL NUMBER OF FIRINGS	TOTAL NUMBER OF AIR LAUNCHES	TOTAL 10-YEAR COST (REC) (in millions) Discount Rate 5%	10%
0	0	78.9	67.6
1057	57	114.2	95.6
1500	500	128.7	106.1
2000	1000	144.5	118.7
3000	2000	179.2	145.1
4000	3000	211.9	170.7
		TOTAL 5-YEAR COST (REC) (in millions) Discount Rate 5%	10%
0	0	31.2	30.2
1057	57	50.2	46.7
2000	1000	69.1	63.2
3000	2000	89.1	80.1

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[1] For periods in which no firings are conducted, the fixed costs associated with maintaining the capability to fire, if required, are incurred. For this reason the total 10-year costs at a zero firing rate are still high.

[2] Appendix B contains a sample cost calculation using 10 per cent discount rate and 10-year planning period.

Let

K_{EXP} = Total amoritized system cost for EXP torpedo for the planning period

K_R = Research and development costs, total (all considered a year one cost)

K_{pj} = Procurement cost per torpedo adjusted for unit learning curve of 85 per cent, j^{th} year.

K_S = Annual spare parts cost per 1000 torpedoes

K_M = Annual test support to insure reliability (fixed cost)

j = Year of the planning period

m = Planning period length

r = Interest rate

x = Number of total firings annually (air and surface)

The cost equation for the EXP becomes:

$$K_{EXP} = \sum_{j=1}^{m} \frac{1}{(1+r)^j} \left(K_R + K_M + \frac{xK_S}{1000} + xK_{pj} \right)$$

In order to compute a torpedo procurement cost, an additional assumptions must be made. Cost savings can be expected in the system designed for a single operation only over the cost of similar systems designed for repeated usage due to a general lessening of tolerance, performance, and failure time specifications. Such savings which might be available by major torpedo redesign are not considered, but savings available without major redesign are estimated in Table 2.

TABLE 2

REC and EXP PROCUREMENT COSTS

	REC (Cost per Unit)	EXP (Cost per Unit)
Exercise Head	\$ 3,500	\$ 100
Main Assembly	7,500	6,500
TOTAL	\$11,000	\$ 6,600

A total savings per torpedo is estimated to be \$4400 and the EXP cost per torpedo becomes \$10,600. This figure is adjusted for a unit learning curve of 85 per cent to compute the K_{pj} values. Table 3 gives aggregate cost levels for the EXP at various expenditure rates, planning periods and discount rates.

As expected for the EXP, operating costs fell markedly due to the absence of a recovery vessel requirement, associated manpower costs and refurbishment costs. Moderate at low expenditure levels but dominant at higher ones were torpedo procurement costs, again as might be expected.

TABLE 3[1]

EXP TORPEDO COST DATA FOR 5 AND 10 YEAR PLANNING PERIOD AND 5 PER CENT AND 10 PER CENT DISCOUNT RATE

TOTAL NUMBER OF FIRINGS	TOTAL NUMBER OF AIR LAUNCHES	TOTAL 10-YEAR COST (EXP) (in millions)	
		Discount Rate 5%	10%
0	0	11.2	9.4
1057	57	79.1	63.6
1500	500	105.7	86.1
2000	1000	136.5	110.9
3000	2000	198.6	161.1
4000	3000	270.9	212.2
		TOTAL 5-YEAR COST (EXP) (in millions)	
		Discount Rate 5%	10%
0	0	6.8	6.4
1057	57	46.4	41.1
2000	1000	84.2	74.9
3000	2000	122.6	108.7

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[1] Appendix B contains a sample cost calculation using 10 per cent discount rate and 10 year planning period.

At this point in the analysis a new alternative is very apparent. With the present stockpile of REC torpedoes and with recovery cost of DD fired torpedoes zero, a joint usage is possible. The REC torpedoes presently available could be used by surface vessels only while the EXP type could be used by air units. The cost equation is a combination of the first two cost equations and is costed in Table 4 below.

TABLE 4

JOINT USAGE COST DATA

TOTAL NUMBER OF FIRINGS	NUMBER OF AIR LAUNCHES	TOTAL COST (JOINT USAGE) in millions			
		10-YEAR		5-YEAR	
		5%	10%	5%	10%
0	0	69.0	58.7	24.7	24.2
1057	57	105.2	86.3	49.5	45.3
2000	1000	165.	134.8	85.7	76.8
3000	2000	228.4	186.5	126.1	112.7

This joint alternative, however, does not appear as a satisfactory alternative from the curve of Total Cost vs Total Firings, Figure 1. The costing of the joint alternative is handicapped due to the fact it is saddled with the disadvantages of both pure alternatives and denied the advantages of either. Although the joint alternative does not have the TR boat costs, these costs are quickly dominated by the refurbishment and attrition costs of the REC and the high procurement of the EXP. It is only slightly less costly than the present REC system up to slightly more than the base number of surface firings (1000); that is, at about 300 air launchings per year it becomes the most costly possible alternative, and though initially investigated, was discarded as a possible alternative due to the relative dominance of the pure type alternatives. This is demonstrated in Figure 1 below for a 10-year planning period and a 10 per cent discount rate.

Figure 2 shows the Total Cost vs Total Firings curves for the EXP and REC torpedoes at both 5 and 10-year planning periods for a 10 per cent discount rate. Figure 3 shows these same curves at a 5 per cent discount rate.

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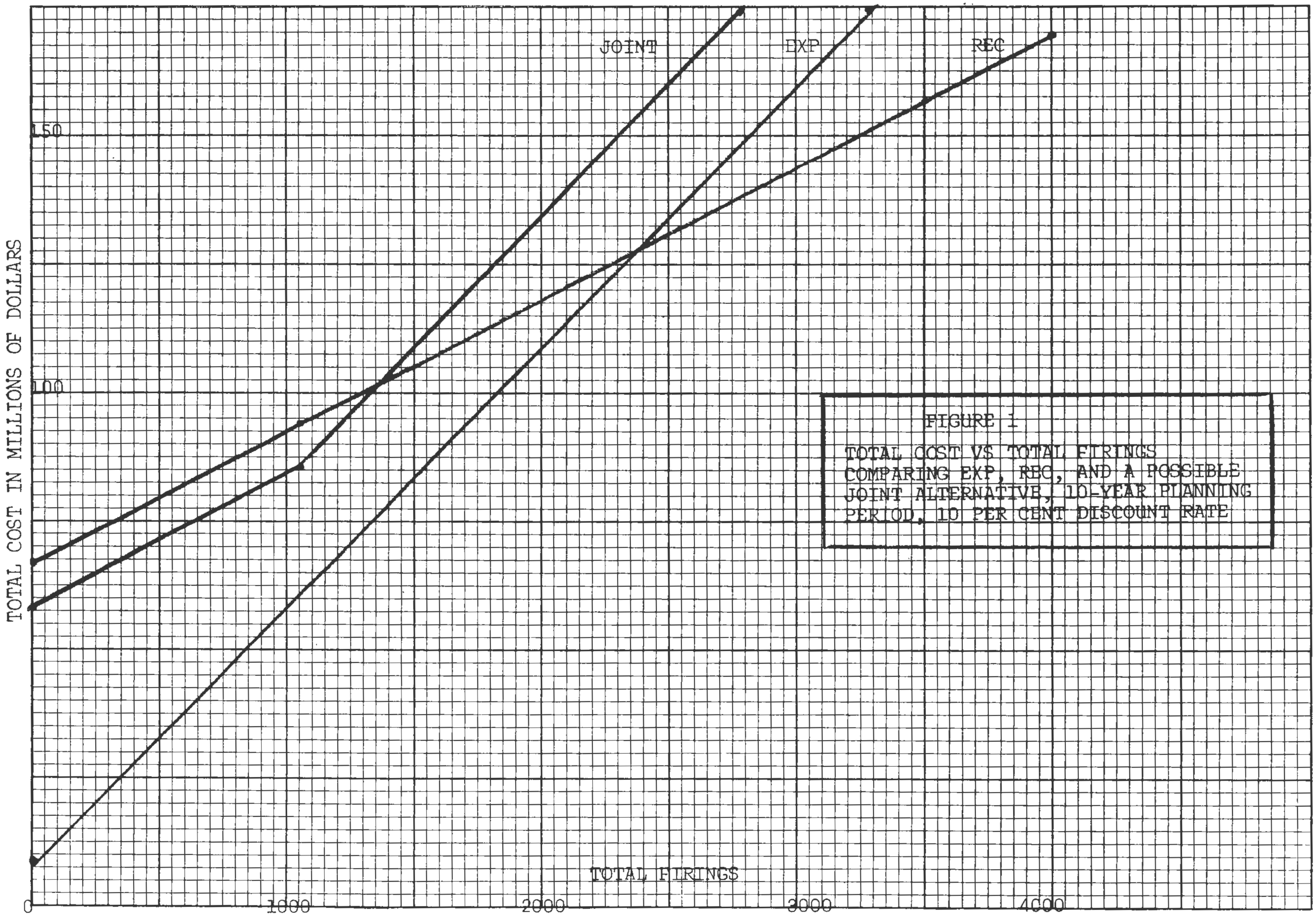

FIGURE 1

TOTAL COST VS TOTAL FIRINGS COMPARING EXP, REC, AND A POSSIBLE JOINT ALTERNATIVE, 10-YEAR PLANNING PERIOD, 10 PER CENT DISCOUNT RATE

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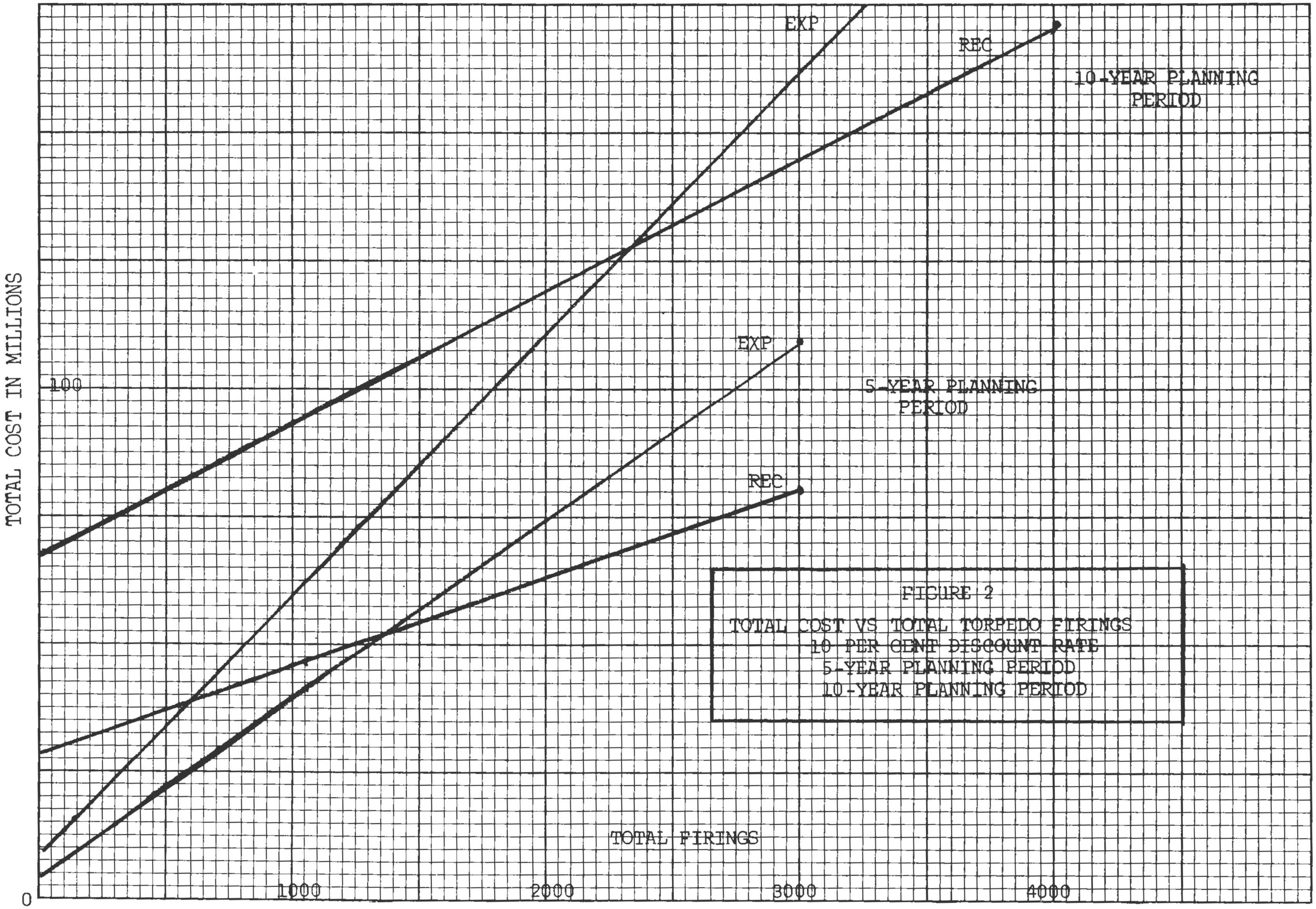

FIGURE 2

TOTAL COST VS TOTAL TORPEDO FIRINGS
10 PER CENT DISCOUNT RATE
5-YEAR PLANNING PERIOD
10-YEAR PLANNING PERIOD

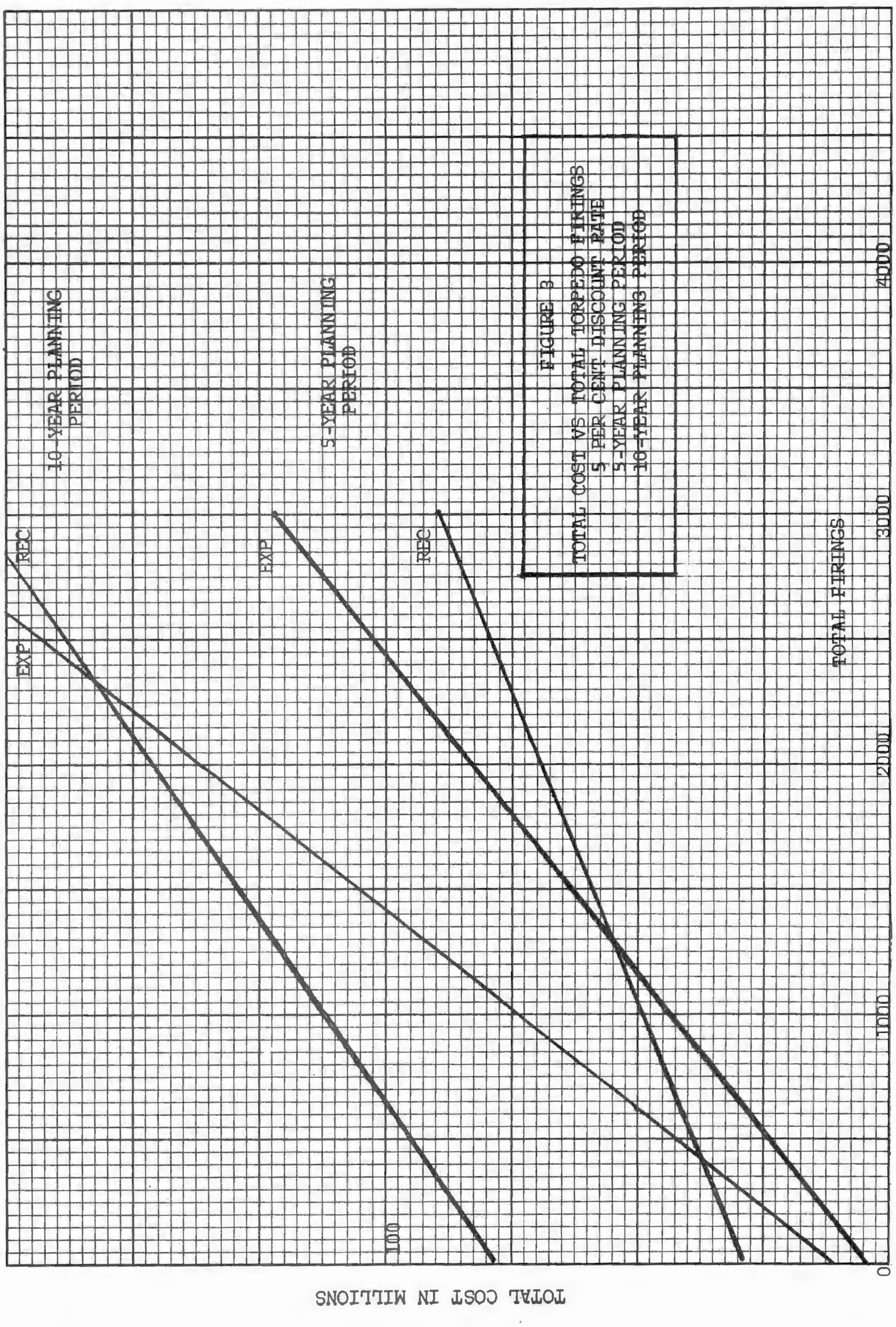

FIGURE 3

TOTAL COST VS TOTAL TORPEDO FIRINGS
5 PER CENT DISCOUNT RATE
5-YEAR PLANNING PERIOD
10-YEAR PLANNING PERIOD

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CHAPTER V

RESULTS OF THE ANALYSIS

Study Results

It is evident that further work must be done in the derivation of the production function for the torpedo problem. The issues of incomplete data, misspecification, omitted variables, and multi-collinearity remain and without additional data and investigation leave the validity of the production function open to question. The hypothesis of the relationship between readiness scores and number of torpedo drops has not been proven or disproven and further investigation with more complete data is called for.

In order to continue the analysis and determine the efficiency problem presented, it is first necessary to select a best production function from those available. Of the regressions computed, run number 261 is selected for a number of reasons. Although it does not have the highest R^2 or $\bar{R}^2$, the R^2 and $\bar{R}^2$ values are satisfactory compared to the other equations. Intuitively, the dummy variables are desirable to counter the bias resulting from squadron type and location evident from the raw data. Variables X_6 and X_7 which did not demonstrate significance are not involved in the equation. The equation, also, is in a total log form which seems intuitively to fit the type of data expected. Run 261 has the additionally attractive feature of similarity of form to the

Cobb-Douglas production function, so that the coefficient of X_9 can be considered an elasticity with respect to the torpedo drop input. The X_1 through X_5 variables are dummy variables and in effect are some constant depending upon the type squadron and its geographic location. The X_9 coefficient shows that marginal productivity falls as input grows. There is no asympototic level of output beyond which output cannot grow (such as 100% readiness), but the rate of increase decreases at high levels of input. Hence Run 261 has a constant elasticity of output variation with regard to the input of the number of torpedo launches. Runs 261, 137 and 221 all give reasonably similar production functions, total cost and marginal cost curves. Run 261 appears to be the best of these three and will be used as the production function for the analysis.

This production function is in reality a group of five curves of identical slope[1] for each of the five data bases (VPP, VPL, VSL, VSP and HSL).[2] The production function is shown in Figure 4. For clarity of presentation only one of the data base production function curves will be used for the remainder of the analysis, that of VPP, but it must be borne in mind that the analysis will be equally valid for all of the data bases. They are images of one other shifted to higher readiness levels.

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[1] Although the five curves do not have precisely the identical slopes, they are identical for all practical purposes. The maximum slope difference is computed in Appendix C.

[2] The reader may wish to refer to page 15 where these data bases are defined.

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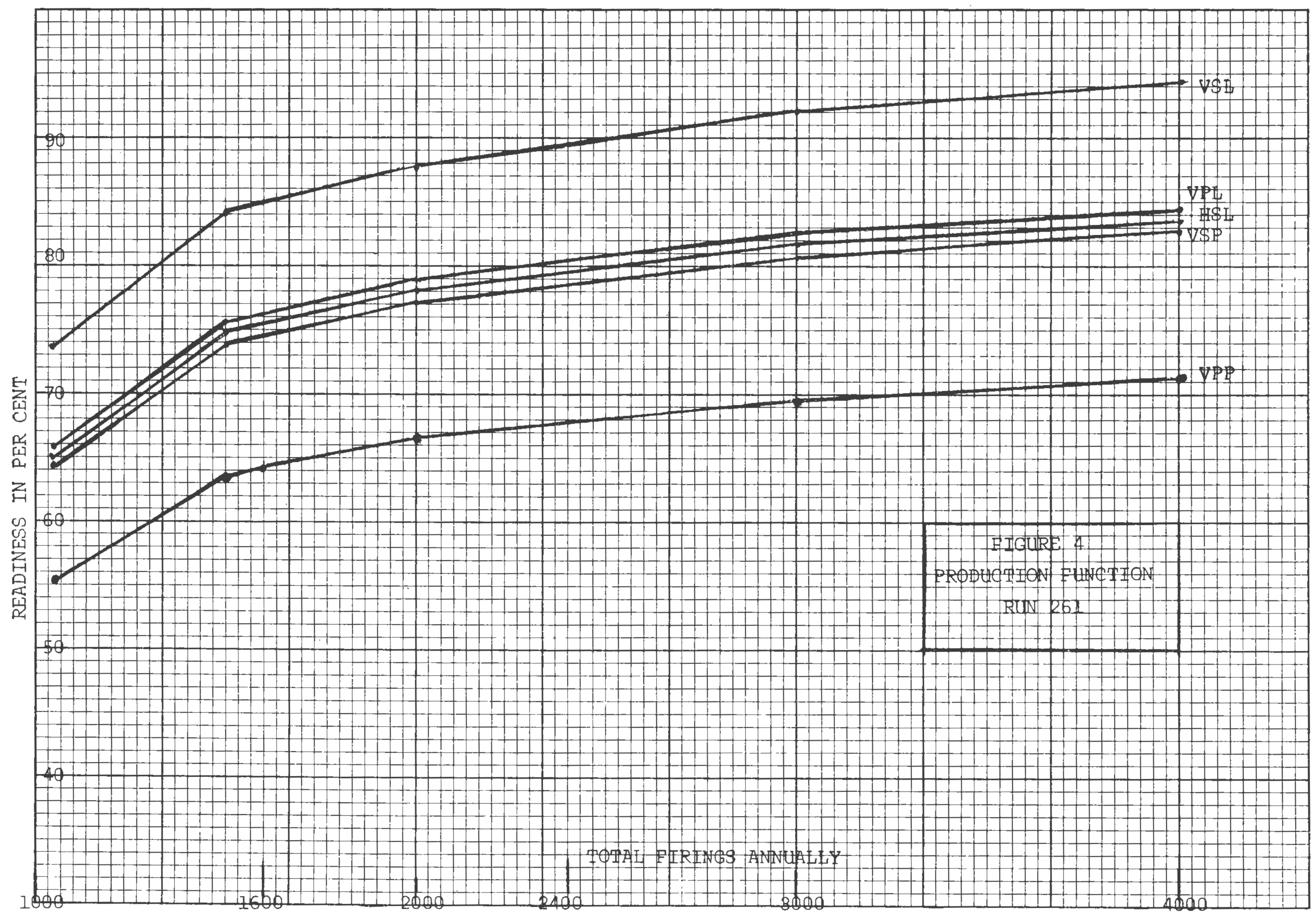

FIGURE 4
PRODUCTION FUNCTION
RUN 261

With this effectiveness function and the two cost functions derived in Chapter IV, we are now ready to construct the total cost and marginal cost curves which were the objectives of the analysis. Table 5 is a compilation of readiness and cost data for the EXP and REC for a 10 year planning period and 10 per cent discount rate. The cost data is taken from Tables 1 and 3. The corresponding readiness levels for VPP are computed using Run 261 as the effectiveness function. Resulting marginal cost computations are also included.

TABLE 5

VALUES OF TOTAL COST, MARGINAL COST AND READINESS FOR EXP AND REC ALTERNATIVES

Total Firings	Total Cost		Marginal Cost		Per Cent Readiness
	REC	EXP	REC	EXP	
1057	95.6	63.6	-	-	55.5
1500	106.1	86.1	1.4	3.0	63.5
2000	118.7	110.9	4.2	8.3	66.5
3000	145.1	161.1	8.5	16.2	69.6
4000	170.7	212.2	15.1	30.0	71.3

Using the data of Table 5, Figure 5 can be constructed. Figure 5 is the Total Cost vs Effectiveness curve for VPP for the same 10-year Planning Period and 10 per cent Discount Rate and is one of the initial comparisons sought in the analysis.

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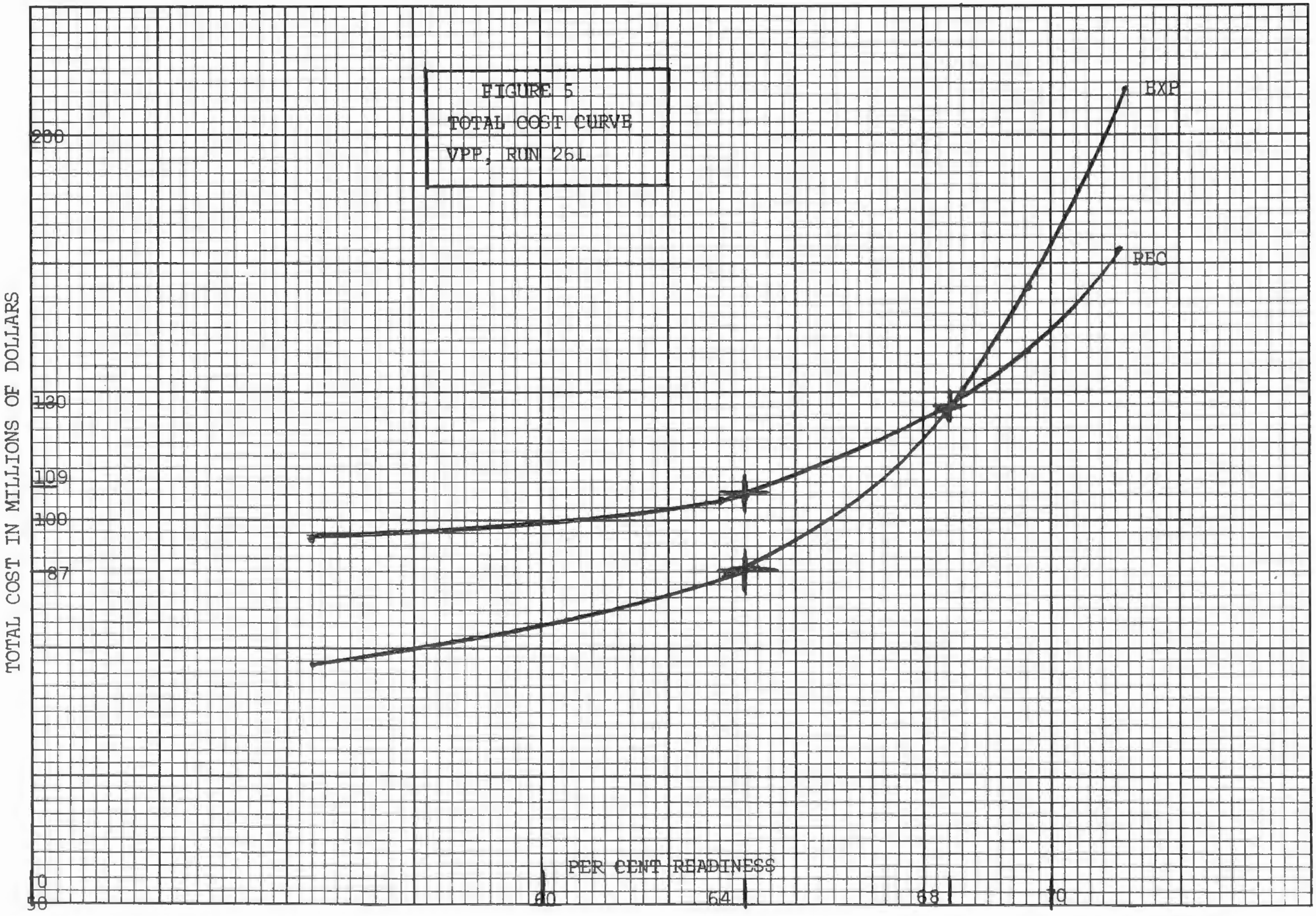

FIGURE 5
TOTAL COST CURVE
VPP, RUN 261

From Table 5 may also be constructed a total cost vs input curve which is shown as Figure 6. The EXP is the least cost system up to a total torpedo drop rate of approximately 2400 torpedoes a year. With the study assumption of 1000 surface launches, this would be equivalent to 1400 air-launched torpedoes annually which is approximately twice the maximum annual air-launch rate observed during the period of the data for the study (through 1966). The Navy is at the present time operating in the range of the production function in which the EXP concept is least costly. To demonstrate this let us take as an example an annual air launch rate of 600; that is, 1600 total torpedo drops annually. This expenditure rate for VPP squadrons corresponds to a readiness score (average) of 64 per cent from Figure 4. To increase the torpedo expenditure level enough to justify the REC concept would require an increase in drop rate by another 800 air launches per year. This would correspond to an increase in average VPP squadron readiness from 64 per cent to 68 per cent, a 4 per cent readiness increase at an increased cost of $130 million less $87 million or $43 million dollars (Figure 5). It must be noted that the increased cost is for an increase in peacetime readiness of 4 per cent for all U.S. Navy ASW squadrons, not just for VPP squadrons. Since the Navy is actually operating under the REC concept, it is paying approximately $109 million for the torpedo launch input to maintain present levels of average squadron readiness (for VPP = 64%). If the objectives of the decision-maker are met by maintaining readiness at present levels,

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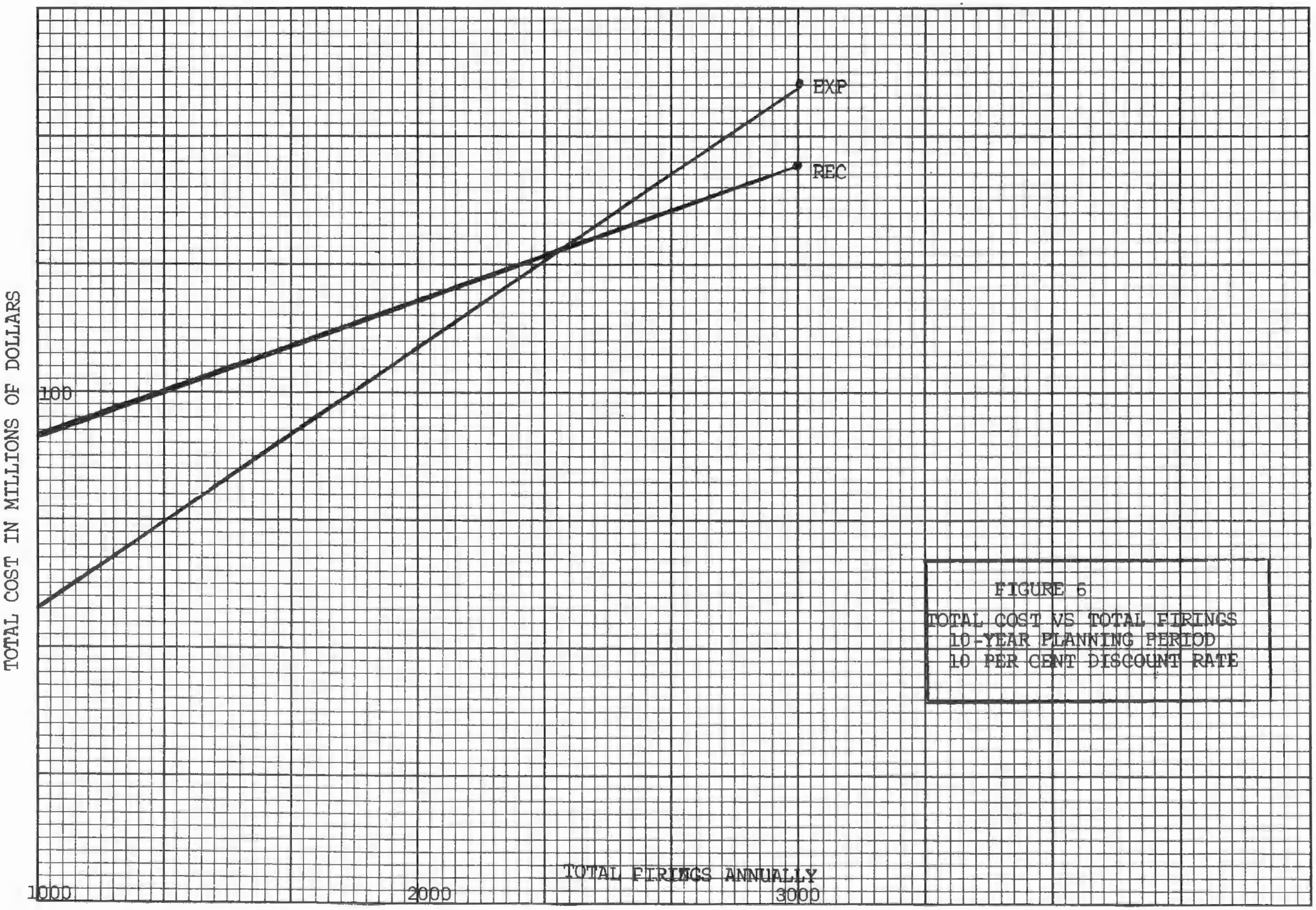

FIGURE 6

TOTAL COST VS TOTAL FIRINGS
10-YEAR PLANNING PERIOD
10 PER CENT DISCOUNT RATE

a saving of $22 million ($109 less $87 million) would be realized by shifting to an expendable torpedo concept.

It is equally possible that the decision-maker may face a budget constraint and be bound by a dollar input rather than a desired level of output. Should he be bound by the approximate present expenditure level of $109 million, the decision-maker can determine his most efficient alternative from the total cost curves of Figure 5. At this cost he can expect a readiness level of 64 per cent using the REC torpedoes or an increased level of 66.5 per cent with the EXP.

If the objectives of the decision-maker indicate that some future readiness increase is required, he must realize that the EXP concept is no longer an attractive alternative after readiness increases of about 4 percentage points above present levels. Up to 68 per cent the EXP concept is the least costly. With a current readiness level of VPP of approximately 64%, it can be seen that a 4 per cent increase in readiness makes the EXP no longer least costly. So if the decision-maker must hedge against possible future increases in required readiness levels, he must be aware of that point where he becomes indifferent between these two alternatives. However, such an across-the-board increase in peacetime squadron capability as 4 percentage points is a significant change in the readiness posture of fleet air units. The significance of even small changes will be made clear in the following paragraphs.

Another important and useful result of the analysis is to provide data to the decision-maker for the evaluation of trade-offs between cost and effectiveness in the solutions of higher level options or alternatives than are addressed by the analysis itself. As we have determined from our total cost curves, increases in readiness come at some predictable cost. To perform a specified mission, fewer squadrons are required if they are all at a high state of readiness than would be required if all were at some lower state. That is, we can accomplish some given objective either with x number of units at a high readiness level or by some number of units less than x if the readiness level is increased for any reason. A cost is incurred to achieve the increase in readiness levels. A saving results from the permissible reduction in force levels. These costs and savings can be compared to determine if the mission objective can be accomplished by a new force of fewer but more effective units at a reduction in total system cost.

This is best illustrated by a hypothetical example. Let us suppose that the 57 ASW squadrons in the Navy are all operating at a readiness level of 60 per cent. (We are taking one production function and one total cost curve to represent all squadrons for the sake of simplicity). Further, let us assume that each squadron has a maximum flight rate of 17.5 sorties per time unit and the ASW sqaudrons have been assigned a mission (by the decision-maker) of flying 600 successful sorties per time unit over an extended period. In order to fly these 600 successful sorties, the squadrons must fly 1000 total sorties since their readiness level of .6 represents their ability to perform this mission. The total required sortie rate raises each squadron to its maximum individual sortie rate of 17.5 missions per time unit $\left(\frac{1000}{57} = 17.5\right)$.

What alternative method exists for completion of the mission with this same type of air unit? The alternative is to increase the readiness level of the air units so that the same output can be maintained with fewer air units.

Let us suppose that, by increase expenditure level of torpedoes (either EXP or REC types), we raise the level of squadron readiness to 61% for some finite cost.

The number of sorties required at the increased readiness level for 600 successful missions falls from 1000 to 984 $\left(\frac{600}{.61} = 984\right)$. In order to fly this number of sorties at the maximum possible rate of 17.5 sorties per squadron per unit time, only 56 squadrons would be required instead of 57 $\left(\frac{984}{17.5} = 56\right)$. Now it becomes an option of the decision-maker to decide whether by raising the level of readiness by some finite amount he can be equally effective in the accomplishment of some higher level requirement and at the same time reduce total system cost by cutting the number of squadrons required to perform the mission.

The other comparison sought at the beginning of the analysis is the Marginal Cost Curves. This marginal cost vs effectiveness is plotted in Figure 7 using the data of Table 5 for VPP at a 10-year Planning Period and a 10 per cent Discount Rate.

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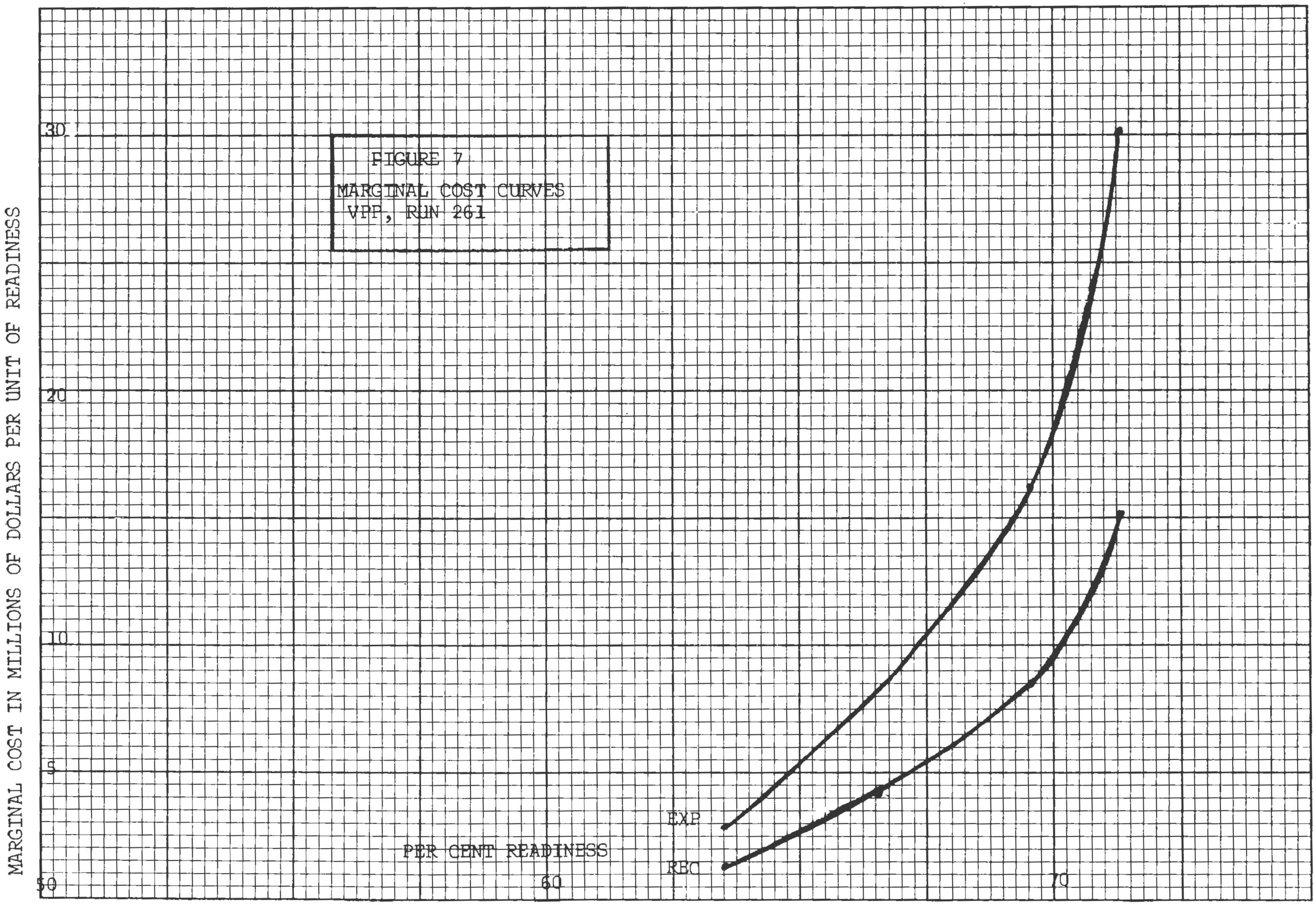

FIGURE 7
MARGINAL COST CURVES
VPP, RUN 261

It can clearly be seen that the marginal cost for the EXP is everywhere higher than the marginal cost for the REC concept. Nevertheless, the EXP is demonstrably the least cost concept for some portion of the output even though it has the higher marginal cost throughout. There is nothing magical here. Marginal cost, being the first derivative or slope of the total cost curve is implicitly shown on the Total Cost curves of Figure 5. It can be seen that for any effectiveness level the EXP total cost curve has a greater slope than the REC total cost curve at the same level of effectiveness. This is the same as saying the EXP has a higher marginal cost throughout. The higher slope does not prevent the EXP total cost curve from being below the REC curve as it is in fact in the lower levels of effectiveness in Figure 5. This demonstrates a pitfall of such ratio comparisons as marginal cost and that to make a decision on marginal cost values alone would be deceptive. Figure 7 shows the decision-maker the incremental cost (saving) for increases (decreases) in readiness.

Sensitivity

One final step in the analysis remains: A sensitivity study of the cost analysis to determine the sensitivity of the results to discount rate, length of planning period and basic torpedo cost.

Using the planning period of ten years, a variation of the discount rate from 10 per cent to 5 per cent does not affect the results of the study; the break-even or cross-over input (total torpedo firing rate) is unchanged at approximately 2400. Total systems cost for both concepts are proportionally higher with the lower discount rate and may throw some doubt on the desirability of any torpedo drop level above 0 due to the higher total cost.

By varying the planning period from 10 to 5 years a very definite change in the results is evident. At this planning period length (at either 10 per cent or 5 per cent discount rate) the cross-over point for indifference between the two alternatives falls from the level of approximately 2400 to approximately 1400 total firings which is equivalent to 400 air launches. At any launch rate above this rate of 400 air launches, the REC concept would be the least costly system. Since the present firing rate is above this range, if the decision-maker anticipates either maintaining or increasing squadron readiness capability he should favor under these assumptions, the REC concept. Although the analysis is critically dependent upon the planning period length selected and a short period favors the existing REC system, from past experience the 10-year planning period is a quite reasonable one as torpedo service life has in the past been generally close to or well above ten years.

What happens to the study results when the value of the REC torpedo is varied? This is a most significant issue, particularly in view of the escalating costs prevalent in defense procurement and the increasing technology in all phases of armament with the resulting cost increases. Although the base cost of $15,000 is a realistic figure for the present air weapon, future weapons may cost much more. The analysis, as might be anticipated, is extremely sensitive to torpedo cost. As the torpedo costs rise, the REC concept retains its advantages over a larger spectrum of the readiness scores. This sensitivity to basic torpedo cost is most appropriately demonstrated on the Total Cost vs Effectiveness curves of Figure 8. While the total costs of the REC remain relatively stable even with a tripling of basic torpedo costs, the EXP cost, both total and marginal climbs rapidly. By doubling the basic torpedo cost, the REC concept dominates at all desired readiness levels. The marginal cost of the EXP rises substantially with increases in the basic torpedo cost as might be anticipated.

Hence once the practice torpedo takes on a cost of roughly twice the cost of the present practice weapon, the recoverable concept dominates at all readiness levels and corresponding torpedo firing rates. Only if the cost of the practice weapons can be retained near the $15,000 cost level does the EXP concept appear as an attractive alternative.

Summary

The problem presented at the outset of the study was to determine what the present concept of recoverable torpedoes offers in terms of cost over the spectrum of effectiveness levels compared to what the concept of an expendable torpedo might offer over the same range. This has been accomplished and presented in Figures 5 and 7 as the Total Cost and Marginal Cost curves under the conditions specified by the assumptions of the study. These results along with the sensitivity tests provide to the decision-maker the necessary information from which he can draw a number of meaningful observations.

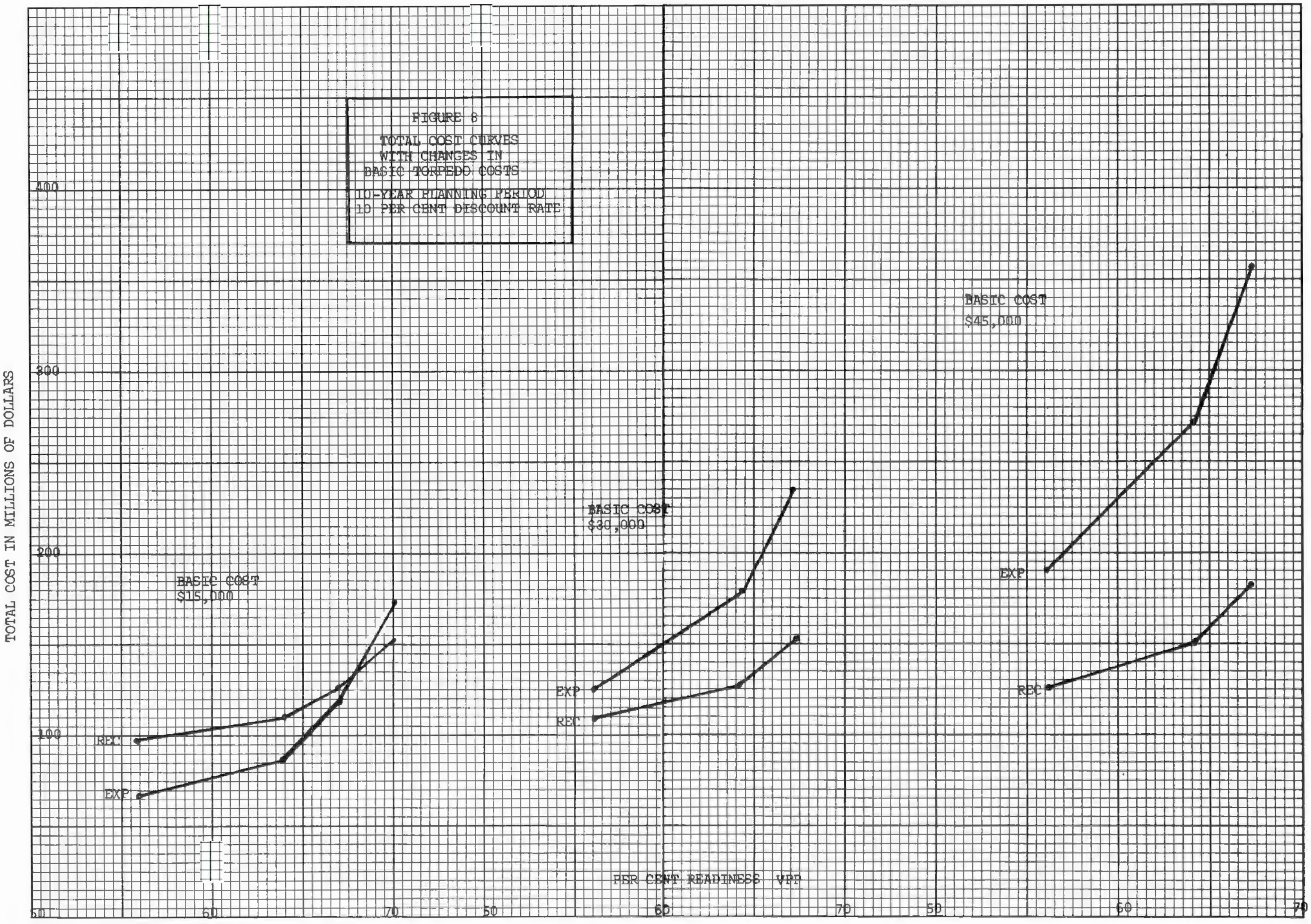

FIGURE 8

TOTAL COST CURVES
WITH CHANGES IN
BASIC TORPEDO COSTS

10-YEAR PLANNING PERIOD
10 PER CENT DISCOUNT RATE

K+E 10 X 10 TO THE INCH 46 0703
7 X 10 INCHES MADE IN U.S.A.
KEUFFEL & ESSER CO.

KEUFFEL & ESSER CO.

1. At the present levels of Readiness and torpedo expenditure rate the EXP concept is less costly an alternative than the REC.
2. If present readiness levels are expected to be maintained or permitted to decline, the EXP concept is a less costly alternative. If, however, the future demand calls for increased training levels and increased ASW squadron readiness levels, then caution must be exercised since the REC becomes the least costly alternative after an increase of squadron readiness levels of 4 percentage points. Such an across-the-board increase in ASW squadron's readiness of 4 per cent is a significant increase.
3. Under conditions of a short planning period, or of high initial torpedo procurement cost, the REC concept dominates the alternatives and the EXP is no longer an attractive alternative.

APPENDIX A

SAMPLE REGRESSION ANALYSIS RESULTS

Following is a table of some of the resulting runs of the regression analysis conducted with the aid of the Harvard Economic Research Project HTW Least Squares Regression Program on an IBM 7094 computer. The three digit index number specifies the run, pass and step numbers of the regression. Dependent and independent variables are listed by subscript and are identified below:

Independent Variables

0 = computed intercept

1 = Pacific-based patrol squadron (VPP) dummy variable

2 = Pacific-based carrier ASW squadron (VSP) dummy variable

3 = Atlantic-based patrol squadron (VPL) dummy variable

4 = Atlantic-based carrier ASW squadron (VSL) dummy variable

5 = Atlantic-based helicopter ASW squadron (HSL) dummy variable

6 = Percent of torpedo hits

7 = $\log_{10}$ of percent of torpedo hits

8 = number of torpedo drops per squadron per year

9 = $\log_{10}$ of the number of torpedo drops per squadron per year

10 = number of torpedo drops per squadron per squadron aircraft per year

11 = log of the number of torpedo drops per squadron per squadron aircraft per year

Dependent Variables

12 = squadron's peacetime readiness score for the year

13 = $\log_{10}$ of squadron's peacetime readiness score

The regression equations listed are in all cases for all data years except for the last two listing which are samples of regressions using just 1964 and 1965 data and are so identified in the table.

RUN NUMBER			DEPENDENT VARIABLE	INDEPENDENT VARIABLE	SLOPE COEFFICIENT	STANDARD ERROR	R^2	$\bar{R}^2$	R	d.f.
Run	Pass	Step								
2	1	1	13	0	1.8192	.01104	.16	.14	.38	98
				8	.00635	.00146				
2	2	1	13	0	1.8075	.014	.14	.13	.35	98
				9	.07637	.01895				
2	3	1	13	0	1.8215	.011	.14	.12	.35	98
				10	.064	.016				
2	4	1	13	0	1.7054	.047	.098	.079	.28	98
				11	.058	.018				
2	4	1	13	1	1.7578	.014	.45	.41	.64	94
				2	1.8199	.013				
				3	1.8288	.013				
				4	1.8761	.013				
				5	1.8238	.018				
				8	.0050	.0012				
2	6	1	13	1	1.7440	.016	.44	.41	.64	94
				2	1.8096	.015				
				3	1.8185	.015				
				4	1.8652	.015				
				5	1.8141	.0195				
				9	.0632	.0157				
2	7	1	13	1	1.759	.0146	.43	.39	.62	94
				2	1.8211	.0144				
				3	1.8308	.0141				
				4	1.8776	.0139				
				5	1.8387	.0170				
				10	.048	.0131				
2	8	1	13	1	1.6442	.039	.43	.39	.63	94
				2	1.7093	.039				
				3	1.7199	.040				
				4	1.7699	.039				
				5	1.7261	.040				
				11	.053	.0146				

RUN NUMBER	DEPENDENT VARIABLE	INDEPENDENT VARIABLE	SLOPE COEFFICIENT	STANDARD ERROR	R^2	$\bar{R}^2$	R	d.f.
117	12	1	57.688	3.08	.47	.43	.66	93
		2	65.787	2.90				
		3	67.685	2.54				
		4	75.710	2.75				
		5	66.819	3.24				
		6	.2943	2.9				
		8	.8642	.187				
118	12	1	58.224	3.06	.49	.45	.67	92
		2	66.596	2.90				
		3	68.462	2.55				
		4	76.702	2.78				
		5	63.679	3.67				
		6	340	2.9				
		8	2.2501	.809				
		10	-15.4	8.7				
127	12	1	56.567	3.57	.47	.43	.65	93
		2	65.214	3.44				
		3	66.902	3.15				
		4	75.006	3.38				
		5	66.155	3.63				
		7	- .6584	1.68				
		9	11.076	2.4				
128	12	1	55.232	11.98	.47	.43	.65	92
		2	63.869	12.02				
		3	65.568	11.86				
		4	73.712	11.59				
		5	64.949	10.95				
		7	- .6673	1.69				
		9	10.391	6.37				
		11	.6810	5.83				

RUN NUMBER	DEPENDENT VARIABLE	INDEPENDENT VARIABLE	SLOPE COEFFICIENT	STANDARD ERROR	R^2	$\bar{R}^2$	R	d.f.
137	12	1	55.087	3.29	.47	.43	.65	93
		2	63.803	3.09				
		3	65.763	2.78				
		4	73.643	2.99				
		5	64.967	3.45				
		6	.606	2.95				
		9	10.826	2.38				
138	12	1	54.091	11.90	.47	.42	.65	92
		2	62.799	11.93				
		3	64.766	11.78				
		4	72.676	11.49				
		5	64.067	10.90				
		6	.59	2.97				
		9	10.311	6.37				
		11	.5092	5.84				
148	12	1	58.717	3.47	.49	.45	.67	92
		2	67.057	3.36				
		3	68.827	3.04				
		4	77.144	2.29				
		5	64.087	3.98				
		7	- .1451	1.64				
		8	2.2493	.80				
		10	-15.35	8.7				
172	12	0	67.844	2.78	.19	.17	.40	97
		6	-1.844	3.4				
		8	1.0589	.223				
173	12	0	67.913	2.80	.19	.15	.39	96
		6	-1.811	3.4				
		8	1.2975	.74				
		10	-2.69	8.				

RUN NUMBER	DEPENDENT VARIABLE	INDEPENDENT VARIABLE	SLOPE COEFFICIENT	STANDARD ERROR	R^2	$\bar{R}^2$	R	d.f.
192	12	0	68.347	6.22	.20	.039	.19	14
		6	5.76	6.9				
(64 data)		8	-2.31	1.30				
117	12	1	62.872	3.49	.68	.60	.77	32
		2	71.669	3.69				
(65 data)		3	67.363	3.77				
		4	82.551	3.68				
		5	71.117	4.27				
		6	- .94	3.3				
		8	.5479	1.19				
		10	-3.3	12.				

APPENDIX B

SAMPLE COST COMPUTATIONS

A sample cost computation for both the REC and EXP will be carried out using a 10 per cent discount rate and a 10-year planning period. Source of the data for the cost computations is the Quarterly Status of Torpedo Firings Reports, 1963, 1964, 1965, 1966, prepared by the U.S. Naval Underwater Weapons Research and Engineering Station, Newport, Rhode Island; as well as interviews with U.S. Naval Ordnance Systems Command, In-Service Torpedo Group personnel.

REC Costing

The REC cost equation is

$$C_{REC} = \sum_{j=1}^{m} \frac{C_{oj} + .2XC_A + .9C_S + .8XC_R + C_D - V_S}{(1 + r)^j}$$

Where

C_{REC} = Total amortized system cost for the REC torpedo for the planning period

C_{oj} = Basic operating cost j^{th} year

C_R = Refurbishment cost per recovered torpedo

C_D = Cost of depot maintenance and spare parts

C_S = Cost of torpedo recovery ships (operating costs and replacement costs combined)

C_A = Attrition cost per torpedo not recovered

V_S = Total torpedo retriever salvage value

x = Number of total firings per year during planning period (air and surface vessels)

j = Planning year

r = Interest rate

m = Length of planning period

Actual computation is significantly simplified by filling in the tableau below:

SAMPLE REC TORPEDO COSTING (1500 Annual Firings)

	Year									
	69	70	71	72	73	74	75	76	77	78
1. Research and Development	zero									
2. Investment										
TR Boats	7.5									
TR Boat Salvage										-3.0
3. Operating Costs										
Depot Main. & Spares	1	1	1	1	1	1	1	1	1	1
Fleet Exercise Support (Proof & Test for 80% Reliability)	9	9	9	9	9	9	9	9	9	9
Refurbishment of 1200 torpedoes	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Torpedo Attrition 300 torpedoes	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
TR Operating Cost	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
TR Replacement Cost	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Torpedo Salvage Value (6000 torpedoes)										-22.5
Total Undiscounted	25.3	17.8	17.8	17.8	17.8	17.8	17.8	17.8	17.8	- 7.7
Discounted 10%	23.1	14.7	13.3	12.1	11.0	10.0	9.1	8.3	7.5	-3.0

$$\sum = \$106.1 \text{ Million}$$

Items in Year 69 will be used to explain the equation. Research and Development costs are considered sunk costs for the REC. C_{oj}, the basic operating costs cover \$9 million for proof and test work to maintain torpedo reliability at 80 per cent as well as fleet exercise support. This includes the costs for refurbishment and repair of extensively damaged exercise torpedoes and such items as periodic testing of inventory torpedoes as well as expenditures for preparation of practice weapons for fleet use. C_{oj}, in addition, includes the \$7.5 million investment expenditure for the TR asset of the force for the first year of the planning period. The element C_D equals \$1 million which is the annual cost for depot maintenance and spare parts attributed to the REC.

The element $.2XC_A$ represents the expected rate of attrition times the number of total firings times the attrition cost per torpedo not recovered. This is $.2(1500)(\$15{,}000)$
$= \$4.5$ million

Similarly the element $.8XC_R$ represents the expected rate of recovery times the firing rate times the refurbishment cost per torpedo recovered. This is $.8(1500)(\$900)$
$= \$1.1$ million

The element $.9C_S$ is determined by the \$1 million annual cost of TR replacement and the basic TR operating cost of \$1.4 million. This is equal to $.9(1.4 + 1.0) = \$2.2$ million.

The final element V_S, the salvage value, is computed using an exponential decay based upon the expected service life of the torpedoes of 10 years. Torpedo inventory is taken to be 9000. Thus it is assumed that the value of the asset at one-half the expected service life is equal to 50 per cent of the original cost, and at the end of the service life the estimated value would be 25 per cent of the original cost. Thus torpedo salvage value is: ¼($15,000)(9000-300(10))

=$22.5 million

Similarly, TR salvage value is: ¼($500,000)(30) ≈ $3 million. These annual costs are then discounted at 10 per cent as shown in the tableau and summed to arrive at the total cost of $106.1 million.

EXP Costing

The EXP cost equation is:

$$K_{EXP} = \sum_{j=1}^{m} \frac{1}{(1+r)^{j}} \left(K_R + K_M + \frac{x\,K_S}{1000} + x\,K_{Pj} \right)$$

Where

K_{EXP} = Total amoritized system cost for EXP torpedo for the planning period

K_R = Research and development costs, total (all considered a year one cost)

K_{Pj} = Procurement cost per torpedo adjusted for unit learning curve of 85 per cent, j^{th} year

K_S = Annual spare parts cost per 1000 torpedoes

K_M = Annual test support to insure reliability (fixed cost)

j = Year of planning period

m = Planning period length

r = Interest rate

x = Number of total firings annually (air and surface)

The actual computation will be simplified as with the REC costing by referring to the following tableau.

SAMPLE EXP TORPEDO COSTING (1500 Annual Firings)

	Year									
	69	70	71	72	73	74	75	76	77	78
1. Research and Development	2.0									
2. Investment										
Test Support for Reliability	1	1	1	1	1	1	1	1	1	1
Maintenance and Spare Parts	.4	.4	.4	.4	.4	.4	.4	.4	.4	.4
3. Operating Costs										
Torpedo Procurement	15.9	13.5	13.5	11.5	11.5	11.5	11.5	9.8	9.8	9.8
Total Undiscounted	19.3	14.9	14.9	12.9	12.9	12.9	12.9	11.2	11.2	11.2
Discounted at 10%	17.7	12.3	11.2	8.8	8.0	7.3	6.6	5.2	4.7	4.3

$$\sum = \$86.1 \text{ Million}$$

The elements of the equation can be explained in the following manner. K_R, the Research and Development Costs are estimated at $2 million and are considered a year one cost. K_M is estimated as $1 million annually while $\frac{x K_S}{1000}$ is computed as follows: $\frac{1500\ (.3)}{1000}$

= $.4 million for the annual cost of maintenance and spare parts.

The final element, xK_{pj}, is the procurement cost for the torpedoes fired each year adjusted for an 85 per cent learning curve. The first year procurement costs are: 1500 ($10,600) = $15.9 million. The learning curve takes place in the second year reducing the procurement cost to $14.5 million (15.9 X .85 = $13.5 million). The next production saving takes place in year four when the procurement cost is again reduced to $11.5 million and likewise it is reduced to $9.8 million in year eight.

These annual cost are then discounted at 10 per cent as shown in the tableau and summed over the planning period to arrive at the total cost of $86.1 million.

APPENDIX C

PRODUCTION FUNCTION, RUN 261

The production function selected for the analysis, Run 261, is a series of five curves for the five data bases. These curves diverge slightly as the value of annual torpedo drops is increased, but for practical purposes the slopes of the five curves are identical.

Run 261 is

$$\log R = 1.744X_1 + 1.8096X_2 + 1.8185X_3 + 1.8652X_4 + 1.8141X_5 + .0632X_9$$

Since X_1 through X_5 are dummy variables and $X_9 = \log X_8$ by definition, Run 261 may be written

$$\log R = K_1X_{(1-5)} + K_2 \log X_8$$

where K_1 = a constant for each curve but may take on five discrete values from 1.744 to 1.8652

$X_{(1-5)}$ = 1 to represent that dummy variable which has the value 1

K_2 = the constant .0632

X_8 = annual torpedo drops per squadron then

$$\log R - K_2 \log X_8 = K_1 \ (1)$$

$$\log \frac{R}{X_8^{K_2}} = K_1$$

$$\frac{R}{X_8^{K_2}} = 10^{K_1}$$

$$R = 10^{K_1} X_8^{K_2}$$

The slope at any point is:

$$\frac{dR}{dX_8} = 10^{K_1} K_2 X_8^{K_2 - 1}$$

Let $K_{1,a}$ and $K_{1,b}$ be two different values of K_1.

The difference in the slope for any two K_1 values is:

$$\Delta \text{ Slope} = \left(10^{K_{1,a}} K_2 X_8^{K_2 - 1}\right) - \left(10^{K_{1,b}} K_2 X_8^{K_2 - 1}\right)$$

$$= K_2 X_8^{K_2 - 1} \left(10^{K_{1,a}} - 10^{K_{1,b}}\right)$$

Substituting the greatest K_1 spread for $K_{1,a}$ and $K_{1,b}$ gives

$$= K_2 X_8^{K_2 - 1} \left(10^{1.8652} - 10^{1.744}\right)$$

$$= .0632 X_8^{.0632-1} \quad (73.31 - 55.46)$$

$$= 1.12 X_8^{-.9368}$$

The higher the value of X_8, the lower the value of the slope difference. The maximum slope difference then will occur at the minimum X_8 value of the production function which is $X_8 = 1$ torpedo fired per squadron per year (1057 total firings annually).

The slope difference is then

$$= 1.12\ (1)^{-.9368}$$

$$= 1.12\ \frac{\text{Degrees of Readiness}}{\text{Torpedoes Fired Per Squadron}}$$

The slope of the lowest production function curve at $X_8 = 1$ is:

$$\text{Slope} = 10^{K_1}\ K_2\ X_8^{K_2 - 1}$$

$$= 10^{1.744}\ (.0632)(1)^{-.9368}$$

$$= 55.46\ (.0632)$$

$$= 3.5\ \frac{\text{Degrees of Readiness}}{\text{Torpedoes Fired Per Squadron}}$$

The slope of the highest production function curve at $X_8 = 1$ is:

$$\text{Slope} = 10^{K_1}\ K_2\ X_8^{K_2 - 1}$$

$$= 10^{1.8652}\ (.0632)(1)^{-.9368}$$

$$= 73.31\ (.0632)$$

$$= 4.62\ \frac{\text{Degrees of Readiness}}{\text{Torpedoes Fired Per Squadron}}$$

This maximum difference in slope rapidly decreases in moving along the X axis. At $X_8 = 10$ firings per squadron (which is equivalent to 1570 total firings annually) the slope of the lowest production function becomes:

$$\text{Slope} = 10^{1.744}(.0632)(10)^{-.9368}$$

$$= 10^{.8072}(.0632)$$

$$= .405 \ \frac{\text{Degrees of Readiness}}{\text{Torpedoes Fired Per Squadron}}$$

The slope of the highest curve is:

$$\text{Slope} = 10^{1.8652}(.0632)(10)^{-.9368}$$

$$= 10^{.9284}(.0632)$$

$$= .535 \ \frac{\text{Degrees of Readiness}}{\text{Torpedoes Fired Per Squadron}}$$

The difference in slope has fallen to $.13 \ \frac{\text{degrees of readiness}}{\text{torpedoes fired per squadron}}$

Differences in slope at greater levels of total firings would be even smaller.

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